

American University of Armenia
Discrete Mathematics
Final Exam
Exam duration: 2
hours

Last Name: _____ First Name: _____

READ THESE INSTRUCTIONS CAREFULLY

- This is a closed-book test, and no notes, assignments, practice problems, books, or other materials are allowed.
- Use of calculators, mobile phones, or any other electronic devices is strongly prohibited. Please turn off your cell phones and place them out of reach.
- Sharing of stationery (pens, pencils, papers, erasers, etc.) is not permitted.
- Talking to another student, looking at another student's paper, or communicating with other students in any way is strictly forbidden.
- Students violating the above rules will be immediately asked to leave the exam room and will get 0 points for the midterm exam.
- You may only leave the room after handing off your papers.
- Use the back side of each page and the last 2 pages of the test booklet to do your draft calculations. Please ask proctors for extra scratch papers if necessary.
- If you run out of space on the test pages, please use a scratch page to finish your work. Indicate in the test page that you will continue on the scratch page, and mark with the rectangle the portion on the scratch page that contains the solution. Any other work on the scratch page will not be graded.
- For the Multiple Choice problem A1, mark only the correct answer by drawing a circle around the label representing that answer. For example, if you find answer (A) to be correct, please mark it as \textcircled{A} .
- Good luck!

DO NOT OPEN THIS BOOKLET
UNTIL YOU HAVE BEEN TOLD TO DO SO

Part A – “Choice” Problems

A1. Circle the label of the (single) correct answer.

24 points

- (a) How many one-to-one function are from $\{1, \dots, n\}$ to $\{1, 2, 3\}$
- (A) n^3 (B) n^{n^2} (C) 3^n (D) 2^n (E) none of the above.
- (b) For two positive integers, we write $m \prec n$ if the sum of the (distinct) prime factors of the first is less to the product of the (distinct) prime factors of the second. For instance, $15 \prec 14$, because $3 + 5 < 2 \cdot 7$. Then is relation \prec is
- (A) not reflexive, anti-symmetric, transitive
(B) reflexive, not anti-symmetric, not transitive
(C) reflexive, anti-symmetric, not transitive
(E) not reflexive, not anti-symmetric, transitive
(D) none of the above.
- (c) Consider a graph G with n edges. What is the biggest number n , for which G and its complement can be planar graphs.
- (A) 13
(B) 14
(C) 16
(E) 17
(D) none of the above.
- (d) Consider the statement, "If n is divisible by k then n is divisible by a and by b and by c ." Which of the following statements is equivalent to this statement?
- A) If n is not divisible by k then n is divisible by c or divisible by b or divisible by a .
B) If n is not divisible by k then n is not divisible by a or not divisible by b or not divisible by c .
C) If n is divisible by a and divisible by b and divisible by c then n is divisible by k .
D) If n is not divisible by a or not divisible by b or not divisible by c then n is not divisible by k .
E) If n is divisible by a or divisible by b or divisible by c then n is divisible by k .
- (e) Professor McBrain has taught the same course for the last 16 years and tells 4 jokes each year. He has never told the same set of three jokes twice (the order of

the jokes is unimportant). How many jokes must he know?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) none of the above.
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A2. True or false? Check the correct box for each question.

6 points

(a) For any 34 people, at least four must be born in the same month.

true false

(b) Function $f : \mathbb{N} \rightarrow \mathbb{N}$, defined as $f(x) = [x] + 1$ is a onto.

true false

Part B – “Show Work” Problems

B1. Prove or disprove:

a) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv (\bar{q} \vee \bar{r}) \rightarrow \bar{p}.$

b) $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

20 points

B2. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

- A) The first 13 letters of the permutation are in alphabetical order.
- B) a is the first letter of the permutation and z is the last letter.
- C) a and b are not next to each other in the permutation.

30 points

B3. Prove or disprove:

A) If a graph is k -regular and k is even, then the number of vertices is odd.

B) If graph is an Eulerian, then its every vertex is non cut vertex.

20 points

B4. Let G be a 6-regular graph with 12 vertices. We are creating a new graph G_1 by adding 3 new vertices to G , and 1 new edge, i.e.

$$|V(G)| + 3 = |V(G_1)|,$$

$$|E(G)| + 1 = |E(G_1)|.$$

Prove or disprove that G_1 is nonplanar.

20 points