A M E R I C A N   U N I V E R S I T Y   O F   A R M E N I A

College of Science and Engineering

Summer Semester, 2018 *Instructor Victor K. Ohanyan*

IESM 106 – Probability and Statistics,

Waived Examination

20 August, 11:30 – 13:20

Student Name ________________________________

Problem 1 (20%)	______________________
Problem 2 (20%)	______________________
Problem 3 (15%)	______________________
Problem 4 (15%)	______________________
Problem 5 (10%)	______________________
Problem 6 (10%)	______________________
Problem 7 (5%)	______________________
Problem 8 (5%)	______________________
Total	______________________

Please write clearly and state any assumptions you make. You can use only ordinary calculators for computation. The use of mobile phones or tablets is strongly prohibited. Please turn off your cell phones.

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PROBLEM 1. An unbiased coin is tossed 8 times.

a. Find the probability that there will be at least one "tails".

b. What is the probability that there will be more "tails" than "heads"?

c. Calculate the probability that a random series either begins with 4 "heads" or ends with 4 "tails"?

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PROBLEM 2. Let η be an absolutely continuous random variable with density function

$$f_\eta(x) = \begin{cases} \frac{1}{2} & \text{if} \quad 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Let $U = -\eta^2$. Find the probability density function of U.

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PROBLEM 3. J&M have their child in daycare twice a week. Being busy people they are often a few minutes late to pick her up. The daycare has a strict policy that parents need to be on time. They enforce this by charging \$1 per minute for tardiness. Suppose that each day the amount of time in minutes that they are late follows an exponential distribution with mean 6. Their child will be in daycare for 100 days this year. Estimate the probability that they will pay more than \$630 in late fees?

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PROBLEM 4. Let the continuous random variables η_1 and η_2 have joint density function

$$f(x,y) = \begin{cases} \frac{1}{x} & \text{if} \quad 0 < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Compute $\operatorname{Cov}(\eta_1, \eta_2)$.

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PROBLEM 5. Recall the relation between degrees Fahrenheit and degrees Celsius

$$\text{degrees Celsius} = \frac{5}{9} \times \text{degrees Fahrenheit} - \frac{160}{9}.$$

Let X and Y be the daily high temperature in degrees Fahrenheit for the summer in Los Angeles and San Diego. Let η_1 and η_2 be the same temperatures in degrees Celsius. Suppose that $\mathrm{Cov}(X, Y) = 5$ and $\rho(X, Y) = 0.9$. Compute $\mathrm{Cov}(\eta_1, \eta_2)$ and $\rho(\eta_1, \eta_2)$ ($\rho(\eta_1, \eta_2)$ = correlation).

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PROBLEM 6. Let A and B be events with probabilities $P(A) = 5/6$ and $P(B) = 1/4$. What is the maximum and minimum values of $P(A \cap B)$. Find corresponding bounds for $P(A \cup B)$.

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PROBLEM 7. We know the expectation of random variable η:

$$E\eta = \frac{1}{\theta}.$$

Find the estimator for unknown parameter $\theta > 0$ by the method of moments, if we have the following sample of size 5:

$X_1 = 2,\ X_2 = 2.5,\ X_3 = 3.5,\ X_4 = 1.5$ and $X_5 = 0.5.$

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PROBLEM 8. The Pareto distribution with parameter α has probability density function

$$f(x) = \begin{cases} \frac{\alpha}{x^{\alpha}} & x \in [1, 8] \\ 0 & \text{otherwise.} \end{cases}.$$

Suppose the data 5, 2, 3 was drawn independently from such a distribution. Find the maximum likelihood estimate of α.